

PATACON

Policy Analysis Tool Applied to Colombian Needs

Departamento de Modelos Macroeconómicos

Banco de la República

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Outline

- 1 Introduction
- 2 Model Features
- 3 Model Structure
- 4 Calibration: General Methodology
- 5 Some Results of the Calibration
- 6 Estimation
- 7 Forecast
- 8 Impulse Response Analysis



References

- González, A., L. Mahadeva, J. D. Prada, and D. Rodríguez (2011): “Policy Analysis Tool Applied to Colombian Needs: PATACON Model Description”, Borradores de economía, 656. Banco de la República.
- Bonaldi, P., González, A. Rodríguez, D. L. E. Rojas, and J. D. Prada (2009): “A numerical method for calibrating a DSGE model”, Borradores de economía, 548. Banco de la República
- González, A., Mahadeva, L. D. Rodríguez, and L.E Rojas (2009): “Monetary Policy Forecasting in a DSGE Model with Data that is Uncertain and Unbalanced”, Borradores de economía, 559. Banco de la República
- Bonaldi, P., González, A. and D. Rodríguez (2010): Importancia de las rigideces nominales y reales en Colombia: un enfoque de equilibrio general dinámico y estocástico”, Borradores de economía, 559. Banco de la República



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The Model

- PATACON is a DSGE model for policy analysis and forecasting of the Colombian economy.
- The model follows Christiano, Eichenbaum and Evans (2005) and adds characteristics to replicate a small open economy. See González, Mahadeva, Prada and Rodríguez (2011).



General Features I: Nominal and Real Rigidities

- Monopolistic competition in labor and goods market with sticky prices á la Calvo (Erceg, Henderson and Levin (2000) and Kollman (1997)).
- Partial price and wage indexation (past inflation).
- Variable capital utilization with endogenous depreciation as a function of capital utilization. (Greenwood et al, (1988), King and Rebelo (1999), CEE (2001)).
- External habit (Abel (1990), Fuhrer (2000)).
- Adjustment costs are in terms of the change in the flow of investment.
- Incomplete exchange rate pass-through (sticky prices á la Calvo and distribution of imported goods).
- Transforming firms



General Features II: External Shocks and Sources of Fluctuations

External factors:

- External demand (prices and quantities).
- Remittances.
- External interest rate.
- Raw materials prices.
- External inflation.
- Prices of imported goods of consumption and investment.

Internal factors:

- Monetary policy shocks.
- Exogenous changes in consumption.
- Exogenous changes in labor supply (intensive margin).



General Features III: Exogenous Shocks and Sources of Fluctuations

Internal factors:

- Exogenous movements in Tobin's Q (investment efficiency).
- Transitory shocks to the productivity in the final good production.
- Permanent shocks to the long run growth of productivity.
- Shocks to the “mark-up” to the final good price and wages.

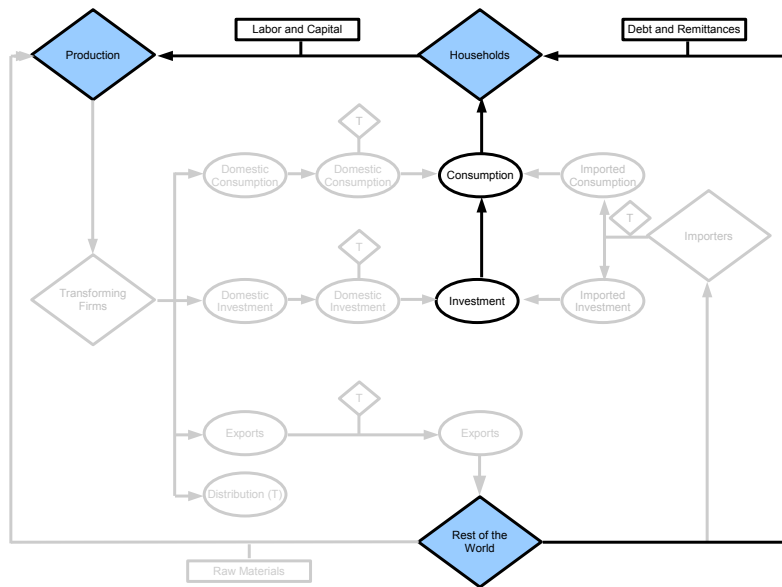


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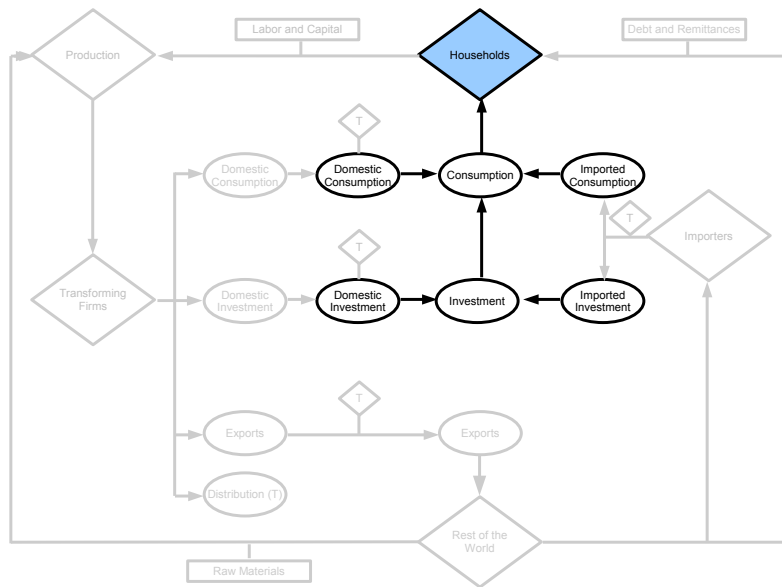
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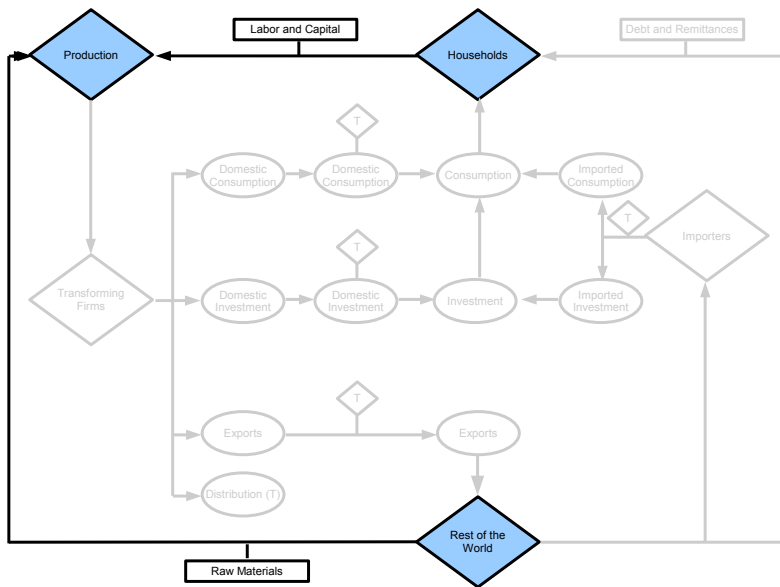
Model Structure 1



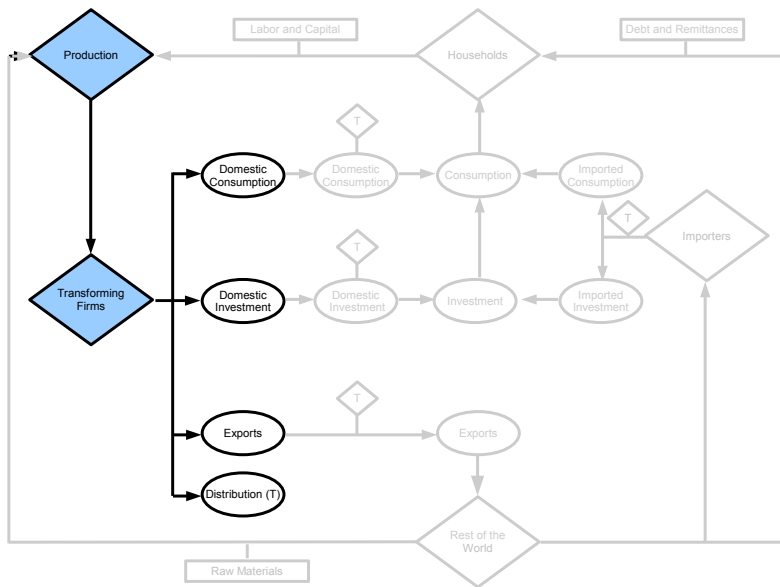
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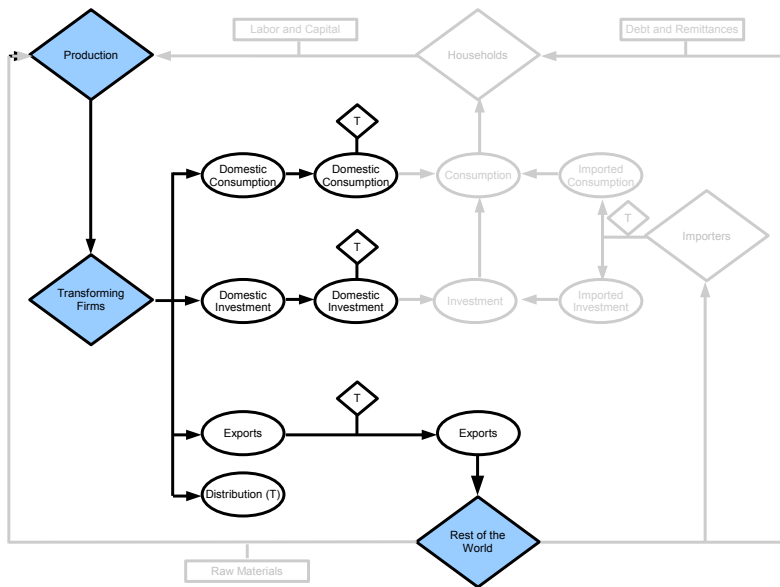
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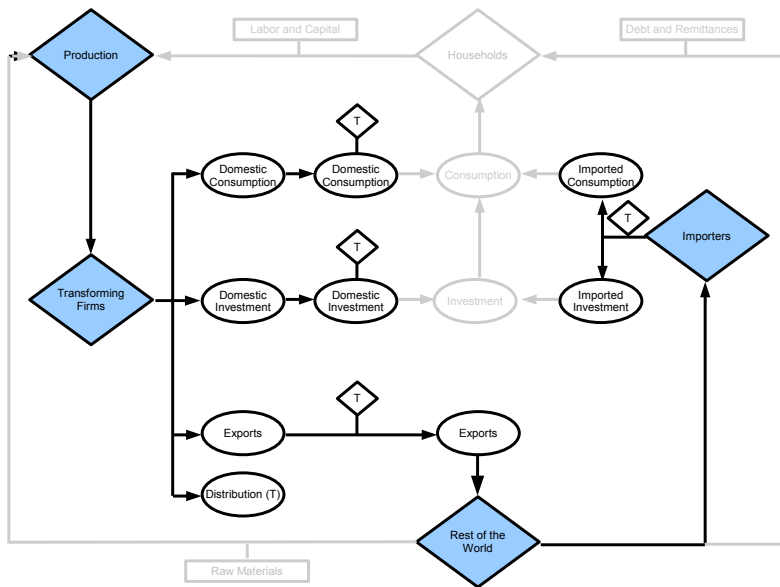
Model Structure 4



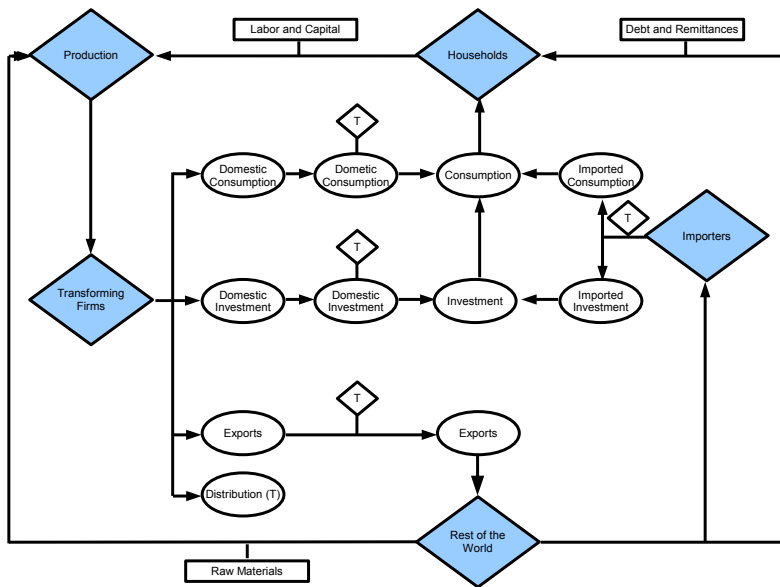
Model Structure 5



Model Structure 6



Model Structure 7



Technological progress, population and unemployment

- Total population N_t follows a process

$$\ln \left(\frac{N_t}{N_{t-1}} \right) = \rho_n \ln \left(\frac{N_{t-1}}{N_{t-2}} \right) + (1 - \rho_n) \ln(1 + \bar{n}) + \epsilon_t^n$$

- Labor force is defined as

$$L_t = (1 - TD_t) TBP_t N_t$$

- Technological progress (in this model equivalent to trend productivity per hour worked) follows:

$$\ln \left(\frac{A_t}{A_{t-1}} \right) = \rho_a \ln \left(\frac{A_{t-1}}{A_{t-2}} \right) + (1 - \rho_a) \ln(1 + g_a) + \epsilon_t^a$$



Model units

- Models such as these are easier to solve if variables can be expressed as stationary, mean zero, deviations for the steady state
- Therefore we express all variables in model units, effectively adjusting them for the two sources of growth, population and Harrod neutral technological progress
- Let J_t in uppercase be the total quantity of a real economic variable, such as the volume of consumption

- ▶ Per-capita terms

$$\tilde{j}_t \equiv \frac{J_t}{N_t}$$

- ▶ Model units

$$j_t \equiv \frac{J_t}{Z_t N_t \bar{l}}$$

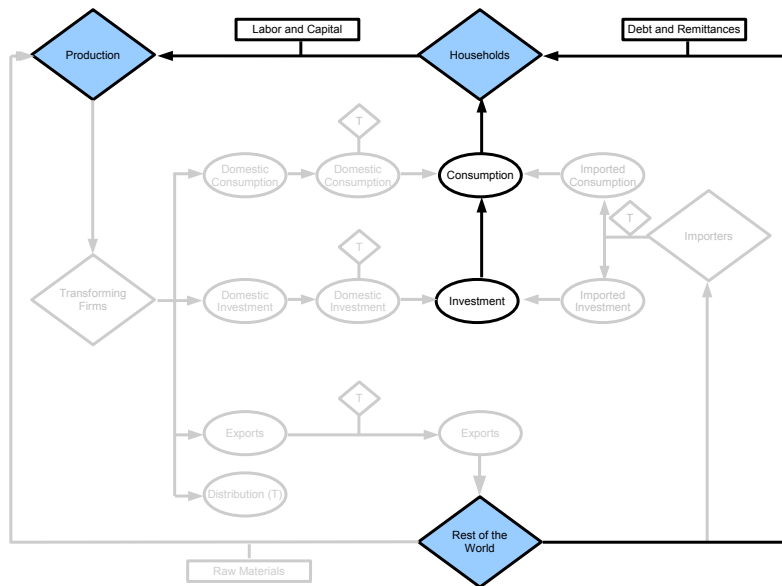
where \bar{l} is total hours available per person and

$$Z_t = Z_{t-1}^{\gamma g} A_t^{1-\gamma g}$$

and $\frac{Z_t}{Z_{t-1}} = (1 + g_t^Z)$



Household



Household

- Continuum of households j of measure one, indexed by $j \in (0, 1)$.
- Utility function

$$u(\cdot) = \left[\begin{array}{l} \left(\frac{z_t^u}{1-\sigma} \left[c_t^F(j) - hab\bar{c}_{t-1}^F \right]^{1-\sigma} \right) \\ - \left(\frac{z_t^h}{1+\eta} \left((1 - TD_t) TBP_t h_t(j) \right)^{1+\eta} \right) \end{array} \right] (Z_t \bar{l})^{1-\sigma}$$

- where $h_t(j)$ the proportion of total hours that are worked.



Household

- Budget constraint

$$\begin{aligned}
 & c_t^F(j) + \frac{p_t^{xF}}{p_t^{cF}} x_t^F(j) + b_t(j) + \frac{s_t p_t^{c^*}}{p_t^{cF}} \frac{1 + i_{t-1}^*}{1 + \pi_t^{c^*}} \frac{b_{t-1}^*(j)}{(1 + \bar{n})(1 + g_t)} \\
 & + \int p_{t+1,t}^a(j) a_{t+1}(j) d\omega_{t+1,t}(j) + \Psi^X(x_t^F(j), x_{t-1}^F(j)) = \\
 & r_t^k u_t(j) \frac{k_{t-1}(j)}{(1 + \bar{n})(1 + g_t)} + w_t(j) (1 - TD_t) TBP_t h_t(j) \\
 & + \xi_t + a_t(j) + \frac{s_t p_t^{c^*}}{p_t^{cF}} tr_t^* + \frac{s_t p_t^{c^*}}{p_t^{cF}} b_t^*(j) \\
 & + \frac{b_{t-1}(j)}{(1 + \bar{n})(1 + g_t)} \left(\frac{1 + i_{t-1}}{1 + \pi_t^{cF}} \right)
 \end{aligned}$$



Household

- Investment cost

$$\psi^X \left(x_t^F(j), x_{t-1}^F(j) \right) = \frac{\psi^X \left(x_t^F(j) - x_{t-1}^F(j) \right)^2}{2 x_{t-1}^F(j)}$$

- Capital accumulation equation

$$k_t(j) = x_t^F(j) + \frac{(1 - \delta(u_t(j))) k_{t-1}(j)}{(1 + \bar{n})(1 + g_t)}$$

- Variable depreciation

$$\delta(u_t(j)) = \bar{\delta} + \frac{b}{1 + \gamma} (u_t(j))^{1+\gamma}$$



F.O.C s

$$\lambda_t^c = z_t^u \left(c_t^F - hab\bar{c}_{t-1}^F \right)^{-\sigma}$$

$$r_t^k = \frac{\lambda_t^x}{\lambda_t^c} b u_t^\gamma$$

$$\lambda_t^c \frac{\rho_t^{x^F}}{\rho_t^{c^F}} = \lambda_t^x - \lambda_t^c \psi^x \frac{x_t^F - x_{t-1}^F}{x_{t-1}^F}$$

$$+ \beta E_t (1 + \bar{n}) (1 + g_{t+1})^{1-\sigma} \lambda_{t+1}^c \left(\frac{\psi^x (x_{t+1}^F - x_t^F) + \Psi^x (x_{t+1}^F, x_t^F)}{x_t^F} \right)$$

$$\lambda_t^x = \beta E_t (1 + g_{t+1})^{-\sigma} \lambda_{t+1}^c r_{t+1}^k u_{t+1} + \beta E_t (1 + g_{t+1})^{-\sigma} \lambda_{t+1}^x (1 - \delta (u_{t+1}))$$

$$\lambda_t^c = \beta E_t (1 + g_{t+1})^{-\sigma} \lambda_{t+1}^c \left(\frac{1 + i_t}{1 + \pi_{t+1}^{c^F}} \right)$$

$$\lambda_t^c = \beta E_t (1 + g_{t+1})^{-\sigma} \lambda_{t+1}^c (1 + i_t^*) \left(\frac{1 + d_{t+1}}{1 + \pi_{t+1}^{c^F}} \right)$$



- Consumption Bundle

$$c_t^F(j) = \left[(\gamma^c)^{\frac{1}{\omega^c}} \left(c_t^{dF}(j) \right)^{\frac{\omega^c-1}{\omega^c}} + (1 - \gamma^c)^{\frac{1}{\omega^c}} \left(c_t^{mF}(j) \right)^{\frac{\omega^c-1}{\omega^c}} \right]^{\frac{\omega^c}{\omega^c-1}}$$

$$c_t^{dF}(j) = \gamma^c \left(\frac{p_t^{cdF}}{p_t^{cF}} \right)^{-\omega^c} c_t^F(j)$$

$$c_t^{mF}(j) = (1 - \gamma^c) \left(\frac{p_t^{mF}}{p_t^{cF}} \right)^{-\omega^c} c_t^F(j)$$



Labor market

- Workers are hired by intermediaries firms, which combine the work effort of individual workers and supply a joint labour input

$$\begin{aligned} \min_{\{h_t(j)\}} \quad & \int_0^1 \tilde{w}_t(j) \tilde{h}_t(j) dj \\ \text{s.t} \quad & \tilde{h}_t^F \leq \int_0^1 \left[\tilde{h}_t(j)^{\frac{\theta^w-1}{\theta^w}} dj \right]^{\frac{\theta^w}{\theta^w-1}} \end{aligned}$$

- F.O.C will imply

$$\begin{aligned} \tilde{h}_t(j) &= \left(\frac{\tilde{w}_t(j)}{\tilde{w}_t} \right)^{-\theta^w} \tilde{h}_t^F \\ \tilde{w}_t &\equiv \left[\int_0^1 \tilde{w}_t(j)^{1-\theta^w} dj \right]^{\frac{1}{1-\theta^w}} \end{aligned}$$



Nominal wages are sticky (Calvo wages)

- Given the demand for their differentiated labour, individuals can set their wages. Each individual is only free to renegotiate a salary when they receive a random signal which arrives every quarter with probability $1 - \varepsilon^w$.
- If wages are not renegotiated they are set by a rule.

$$w_t^{Rule}(j) = w_{t-1}(j) \left(\frac{1 + \pi_{t-1}^{CF}}{1 + \pi_t^{CF}} \right)$$

- If on the other hand, the j^{th} individual receives the signal to renegotiate her wage at period t , that will be set according to:

$$w_t^{opt}(j) = \frac{\theta^w}{\theta^w - 1} \frac{num_t^w(j)}{den_t^w(j)}$$



Nominal wages are sticky (Calvo wages)

- where

$$\begin{aligned} num_t^w(j) &\equiv E_t \sum_{i=0}^{\infty} (\beta \varepsilon^w (1 + \bar{n}))^i \prod_{k=1}^i [(1 + g_{t+k})^{1-\sigma}] \\ & z_{t+i}^h ((1 - TD_{t+i}) TBP_{t+i})^{1+\eta} \left(h_{t+i}^F \left(\frac{w_t^{opt}(j)}{w_{t+i}} \right)^{-\theta^w} \left(\frac{1 + \pi_t^{CF}}{1 + \pi_{t+i}^{CF}} \right)^{-\theta^w} \right)^{1+\eta} \end{aligned}$$

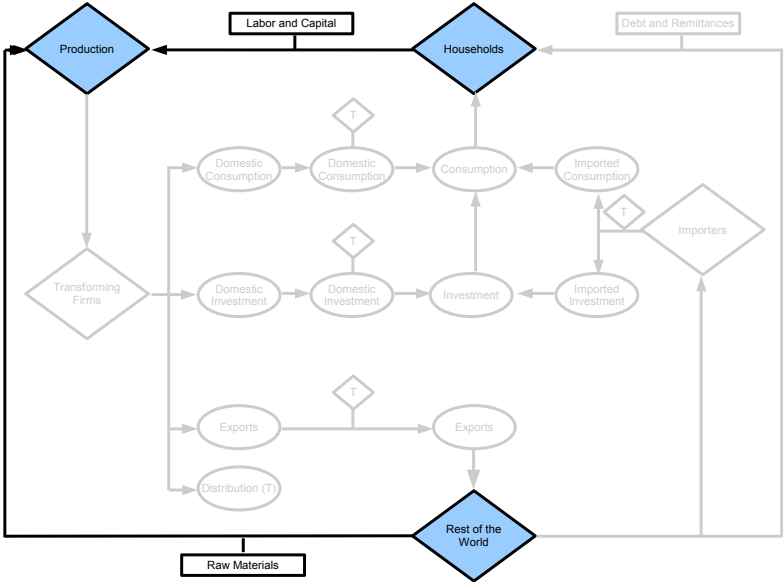
$$\begin{aligned} den_t^w(j) &\equiv E_t \sum_{i=0}^{\infty} (\beta \varepsilon^w (1 + \bar{n}))^i \prod_{k=1}^i [(1 + g_{t+k})^{1-\sigma}] \\ & \lambda_{t+i}^c (1 - TD_{t+i}) TBP_{t+i} \left(h_{t+i}^F \left(\frac{w_t^{opt}(j)}{w_{t+i}} \right)^{-\theta^w} \left(\frac{1 + \pi_t^{CF}}{1 + \pi_{t+i}^{CF}} \right)^{1-\theta^w} \right) \end{aligned}$$

- Then, the real wage evolves according to

$$w_t = \left[\varepsilon^w \left(w_{t-1} \left(\frac{1 + \pi_{t-1}^{CF}}{1 + \pi_t^{CF}} \right) \right)^{1-\theta^w} + (1 - \varepsilon^w) (w_t^{opt})^{1-\theta^w} \right]^{\frac{1}{1-\theta^w}} + z_t^w$$



Intermediate Production Firms



Intermediate Production Firms

- There are a continuum of firms indexed by $z \in (0, 1)$
- Each produces a differentiated product (z) given the following production function which is weakly separable in value-added factors of production

$$q_t^C(z) = z_t^g \left[\alpha^{\frac{1}{\rho}} (va_t(z))^{\frac{\rho-1}{\rho}} + (1-\alpha)^{\frac{1}{\rho}} (rm_t^F(z))^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

$$va_t(z) = \left[\alpha_v^{\frac{1}{\rho_v}} (k_t^S(z))^{\frac{\rho_v-1}{\rho_v}} + (1-\alpha_v)^{\frac{1}{\rho_v}} ((1-TD_t) TBP_t a_t h_t(z))^{\frac{\rho_v-1}{\rho_v}} \right]^{\frac{\rho_v}{\rho_v-1}}$$

- where $a_t = \frac{A_t}{Z_t}$ and $k_t^S = \frac{u_t k_{t-1}}{(1+\bar{n})(1+g_t^z)}$



Intermediate Production Firms

- The first-order conditions are then given as:

$$w_t = \lambda_t^q(z) z_t^q \left(\frac{\alpha q_t^C(z)}{z_t^q v a_t(z)} \right)^{\frac{1}{\rho}} \left(\frac{(1 - \alpha_v) v a_t(z)}{(1 - TD_t) TBP_t a_t h_t(z)} \right)^{\frac{1}{\rho v}}$$

$$r_t^k = \lambda_t^q(z) z_t^q \left(\frac{\alpha q_t^C(z)}{z_t^q v a_t(z)} \right)^{\frac{1}{\rho}} \left(\frac{\alpha_v v a_t(z)}{k_t^S(z)} \right)^{\frac{1}{\rho v}}$$

$$\frac{p_t^{rmF}}{p_t^{cF}} = \lambda_t^q(z) z_t^q \left(\frac{(1 - \alpha) q_t^C(z)}{z_t^q r m_t^F(z)} \right)^{\frac{1}{\rho}}$$

- where $\lambda_t^q(z)$ is the real marginal cost in model units and measured in consumption process of these firms.



Intermediate Production Firms

- The real marginal cost in model units and measured in consumption process of these firms $\lambda_t^q(z)$.

$$\lambda_t^q(z) = \frac{1}{(z_t^q)} \left[\alpha \left(\left[\alpha_v (r_t^k)^{1-\rho_v} + (1-\alpha_v) (w_t)^{1-\rho_v} \right]^{\frac{1}{1-\rho_v}} \right)^{1-\rho} + (1-\alpha) \left(\frac{p_t^{rmF}}{p_t^{cF}} \right)^{1-\rho} \right]^{\frac{1}{1-\rho}}$$

- The price of raw materials in external currency is exogenous

$$p_t^{rmC} = s_t p_t^{rm*}$$



Prices are sticky (Calvo pricing)

- Each period firms face a constant probability $(1 - \varepsilon^q)$ of receiving a signal which tells them when they can adjust their price.
- The other ε^q firms that are not allowed to reset their prices follow a backward-looking indexation rule:

$$p_t^{rule}(z) = p_{t-1}^{qF}(z) \left(1 + \pi_{t-1}^{qF}\right)^{\iota^q} (1 + \bar{\pi})^{1-\iota^q}$$

where $\bar{\pi}$ is average long-run inflation taken to be the central bank's target, $\iota^q \geq 0$ is the weight assigned to past inflation as opposed to this target.



Prices are sticky (Calvo pricing)

- If on the other hand, the z^{th} receives a signal which tells it that can adjust their price, it will choose

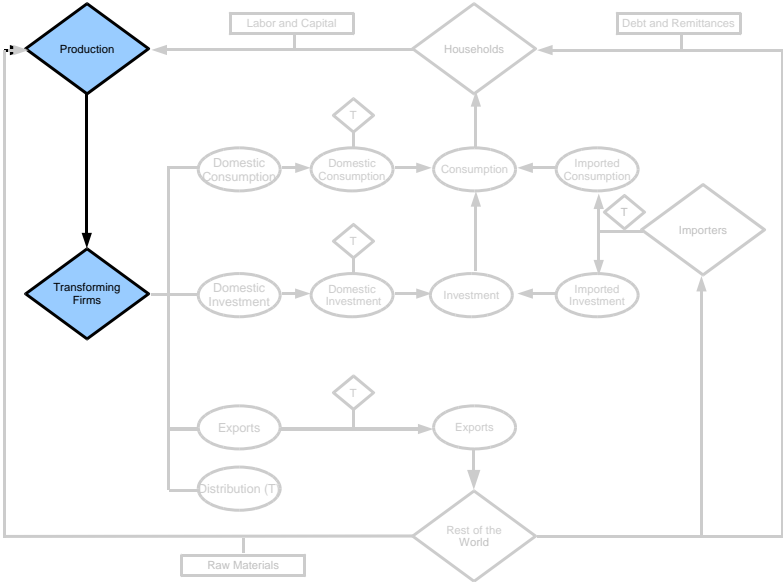
$$\frac{p_t^{qopt}(z)}{p_t^{qF}} = \frac{\theta^q}{\theta^q - 1} \frac{E_t \sum_{i=0}^{\infty} (\varepsilon^q)^i \Delta_{t+i,t} \frac{A_{t+i}}{A_t} \left[\frac{\lambda_{t+i}^q(z) \left(\frac{p_{t+i}^{qF}}{p_t^{qF}} \right)^{\theta^q} q_{t+i}^{qF}}{\left(\prod_{l=1}^i \left\{ (1 + \pi_{t-1+l}^{qF})^{\iota^q} \right\} (1 + \bar{\pi})^{i(1-\iota^q)} \right)^{\theta^q}} \right]}{E_t \sum_{i=0}^{\infty} (\varepsilon^q)^i \Delta_{t+i,t} \frac{A_{t+i}}{A_t} \left[\frac{\left(\frac{p_{t+i}^{qF}}{p_t^{qF}} \right)^{\theta^q - 1} \frac{p_{t+i}^{qF}}{p_{t+i}^{qF}} q_{t+i}^{qF}}{\left(\prod_{l=1}^i \left\{ (1 + \pi_{t-1+l}^{qF})^{\iota^q} \right\} (1 + \bar{\pi})^{i(1-\iota^q)} \right)^{\theta^q - 1}} \right]}$$

- Therefore the inflation dynamics will evolve according with

$$\left(1 + \pi_t^{qF} \right) = \left[\begin{array}{l} (1 - \varepsilon^q) \left(\frac{p_t^{qopt}}{p_t^{qF}} \right)^{1-\theta^q} \left(1 + \pi_t^{qF} \right)^{1-\theta^q} \\ + \varepsilon^q \left[\left(1 + \pi_{t-1}^{qF} \right)^{\iota^q} \left(1 + \bar{\pi} \right)^{i(1-\iota^q)} \right]^{1-\theta^q} \end{array} \right]^{\frac{1}{1-\theta^q}} + z_t^{\pi}$$



Final production good



Final production good

- There is an aggregation technology

$$q_t^F = \left[\int_0^1 \left(q_t^C(z) \right)^{\frac{\theta^q - 1}{\theta^q}} dz \right]^{\frac{\theta^q}{\theta^q - 1}} .$$

- The demand for the intermediate good (z) is given by

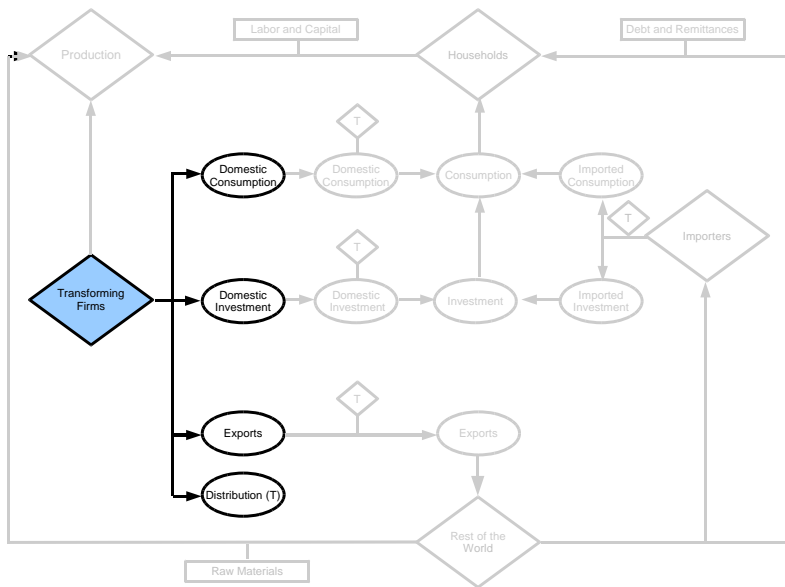
$$q_t^C(z) = \left(\frac{p_t^q(z)}{p_t^{qF}} \right)^{-\theta^q} q_t^F$$

- The output price is given as the aggregate

$$p_t^{qF} = \left[\int_0^1 \left(p_t^q(z) \right)^{1 - \theta^q} dz \right]^{\frac{1}{1 - \theta^q}}$$



Transformation of final good



Transformation of final good

- At a next stage, the final product q_t^F is transformed into four different types of output: domestic consumption, c_t^{dC} , intermediate domestic capital goods, x_t^{dC} , exports, e_t^C , and as distribution services, dis_t^C .

$$q_t^F = \left[\nu_{nt}^{\omega_q-1} (nt_t)^{\omega_q} + \nu_e^{\omega_q-1} (e_t^C)^{\omega_q} \right]^{\frac{1}{\omega_q}}$$

$$nt_t = \left[\nu_c^{\omega_{nt}-1} (c_t^{dC})^{\omega_{nt}} + \nu_x^{\omega_{nt}-1} (x_t^{dC})^{\omega_{nt}} + \nu_{dis}^{\omega_{nt}-1} (dis_t^C)^{\omega_{nt}} \right]^{\frac{1}{\omega_{nt}}}$$



Transformation of final good

- Supplies of each if they are given by

$$\frac{p_t^{cdC}}{p_t^{qF}} = \left(\frac{\nu_{nt} n_t}{q_t^F} \right)^{\omega_{q-1}} \left(\frac{\nu_c C_t^{dC}}{n_t} \right)^{\omega_{nt-1}}$$

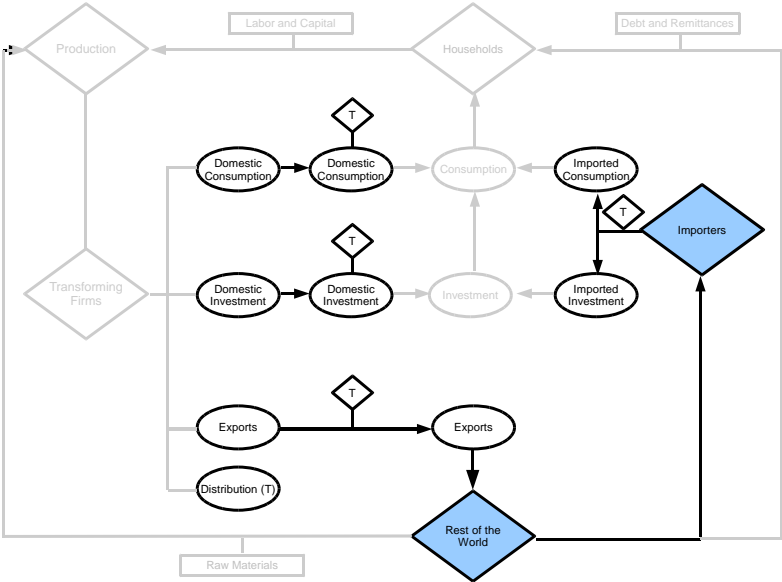
$$\frac{p_t^{xdC}}{p_t^{qF}} = \left(\frac{\nu_{nt} n_t}{q_t^F} \right)^{\omega_{q-1}} \left(\frac{\nu_x X_t^{dC}}{n_t} \right)^{\omega_{nt-1}}$$

$$\frac{p_t^{disC}}{p_t^{qF}} = \left(\frac{\nu_{nt} n_t}{q_t^F} \right)^{\omega_{q-1}} \left(\frac{\nu_{dis} dis_t^C}{n_t} \right)^{\omega_{nt-1}}$$

$$\frac{p_t^{eC}}{p_t^{qF}} = \nu_e^{\omega_{q-1}} \left(\frac{e_t^C}{q_t^F} \right)^{\omega_{q-1}}$$



Distribution



Distribution

- Note that

$$dis_t^F = dis_t^{cd} + dis_t^{xd} + dis_t^e + dis_t^m$$

- Distribution of the domestic product

$$j_t(z) = \left[(\gamma^j)^{\frac{1}{\omega^j}} \left(j_t^C(z) \right)^{\frac{\omega^j-1}{\omega^j}} + (1 - \gamma^j)^{\frac{1}{\omega^j}} \left(dis_t^j(z) \right)^{\frac{\omega^j-1}{\omega^j}} \right]^{\frac{\omega^j}{\omega^j-1}}$$

where j_t : domestic consumption c_t^d , domestic investment x_t^d , exports e_t and imports m_t^* .



Distribution

- Demand for $dis_t^j(z)$

$$\frac{p_t^{jC}}{p_t^{CF}} = \lambda_t^j(z) \left(\frac{\gamma^j j_t(z)}{j_t^C} \right)^{\frac{1}{\omega^j}}$$

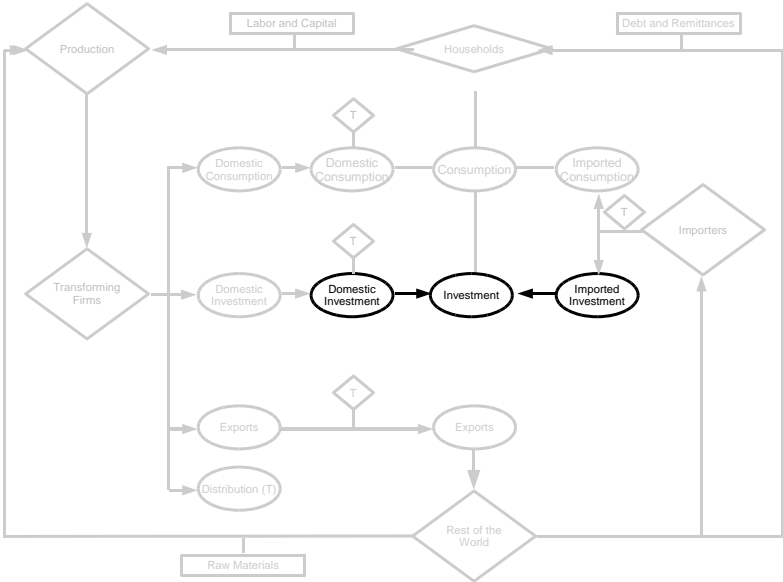
$$\frac{p_t^{disF}}{p_t^{CF}} = \lambda_t^j(z) \left(\frac{(1 - \gamma^j) j_t(z)}{dis_t^j} \right)^{\frac{1}{\omega^j}}$$

- Additionally, each of this sectors is subject to a similar price rigidity as the one described above and the price of import goods is also exogenous

$$p_t^{mC} = s_t p_t^{m*}$$



Investment



- Investment goods produced

$$x_t^F = z_t^x \left[(\gamma^x)^{\frac{1}{\omega^x}} (x_t^{dF})^{\frac{\omega^x-1}{\omega^x}} + (1 - \gamma^x)^{\frac{1}{\omega^x}} (x_t^{mF})^{\frac{\omega^x-1}{\omega^x}} \right]^{\frac{\omega^x}{\omega^x-1}}$$

$$x_t^{dF} = (\gamma^x) \left(\frac{p_t^{xdF}}{z_t^x p_t^{xF}} \right)^{-\omega^x} \frac{x_t^F}{z_t^x}$$

$$x_t^{mF} = (1 - \gamma^x) \left(\frac{p_t^{mF}}{z_t^x p_t^{xF}} \right)^{-\omega^x} \frac{x_t^F}{z_t^x}$$



- External demand

$$p_t^{eF} = s_t p_t^{eE}$$
$$e_t^F = \left(\frac{p_t^{e*}}{p_t^{c*}} \right)^{-\mu} c_t^*$$



Interest rates

- External interest rate

$$i_t^* = \bar{i}^* z_t^{i^*} \exp \left(\Omega_u \left(\frac{s_t \rho_t^{C^*} b_t^*}{\rho_t^{CF} y_t} - \bar{b}^* \right) \right)$$

- Policy rule

$$i_t = \rho_s i_{t-1} + (1 - \rho_s) \left(\bar{i} + \varphi_\pi \left(\pi_t^{CF} - \bar{\pi} \right) + \varphi_y \left(\frac{y_t}{y_t^{flex}} - 1 \right) \right) + z_t^i$$



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General Methodology

- We calibrate the steady state by minimizing the following objective function:

$$f_{obj}(\theta) = \sum_{i=1}^n \omega_i \left(x_i^{ss}(\theta) - x_i^{d-lr} \right)^2$$

where x^{ss} the steady state values and x_i^{d-lr} the equivalent ratios in the data.

- In this exercise we target the 21 nominal ratios.



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- 5 Some Results of the Calibration**
- 6 Estimation
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Results of the Calibration

Ratios and relative prices	Model	Data Colombia	Deviation%
Investment / GDP	0.22	0.23	0.003
Imported investment / Total investment	0.37	0.36	0.030
Domestic inv. without dist. / Gross product	0.13	0.12	0.004
Consumption / GDP	0.82	0.80	0.023
Dom. cons. without dist. / Gross product	0.68	0.60	0.035
Imported consumption / Total consumption	0.12	0.12	0.024
Labor supply	0.30	0.30	0.000
Raw materials / Gross product	0.09	0.10	0.029
FOB Imports / Import. with distribution	0.76	0.73	0.038
FOB imports + raw material / GDP	0.24	0.23	0.025
Exports without dist. / Gross product	0.16	0.17	-0.047



Results of the Calibration

Ratios and relative prices	Model	Data Colombia	Deviation%
Remittances / GDP	0.03	0.04	-0.015
Dom. consumption dist. / Dom. consumption	0.05	0.06	-0.011
Dom. investment dist. / Dom. investment	0.04	0.04	-0.005
Exports dist. / Exports	0.12	0.13	-0.014
Dom. cons. without dist. / Dom. consumption	0.92	0.94	-0.025
Dom. inv. without dist. / Dom. investment	0.93	0.96	-0.026
Exports without dist. / Exports	0.85	0.88	-0.028
Domestic consumption / Gross product	0.68	0.64	0.061
Domestic investment / Gross product	0.13	0.12	0.031
Exports / Gross product	0.19	0.20	-0.019



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Estimation Strategy

- We use bayesian methods to estimate

$$\begin{aligned} \ln P(\theta|y, \tilde{y}) &\propto \underbrace{\ln P(y|\theta)}_{\text{likelihood}} + \underbrace{\ln P(\theta|\tilde{y})}_{\text{prior long-run parameters}} \\ &+ \underbrace{\ln P(\theta)}_{\text{prior short-run parameters}} \end{aligned}$$

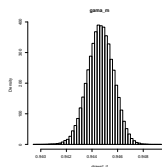
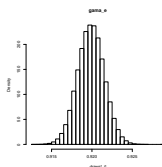
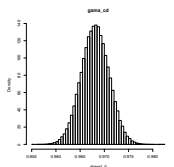
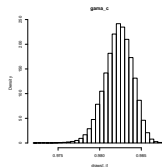
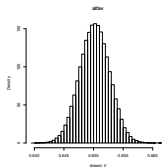
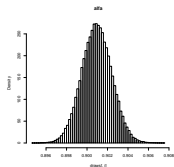
$$\ln P(\theta|\tilde{y}) \sim N(\mu, \Sigma)$$

where μ was set to the parameter values found in the calibration stage. The variance Σ was computed by generating draws from the objective function if the calibration stage.

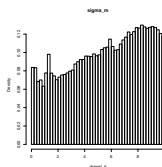
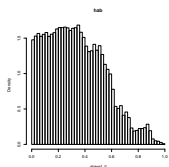
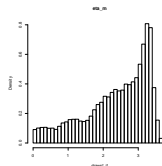
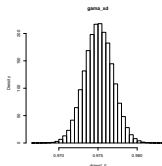
- The priors of the short run parameters are characterized by a large variance.



Draws of the calibration objective function



Draws of the calibration objective function



Estimation

Param.	Dist.	Prior				Posterior		
		Mean	Std	LB	UB	Mode	Mean	Std
η	MN	3.0000	Σ	0.01	5.00	2.475	2.5054	0.31853
hab	MN	0.2100	Σ	0.00	0.99	0.315	0.3160	0.07109
σ	MN	4.0000	Σ	0.01	10.00	4.9	5.0393	0.85355
α	N	0.9008	0.00147	0.10	0.99	0.9011	0.9011	0.00146
α_v	N	0.6501	0.00251	0.10	0.99	0.65025	0.6501	0.00250
γ_c	N	0.9823	0.00169	0.00	1.00	0.9838	0.9838	0.00174
γ_{cd}	N	0.9679	0.00290	0.01	0.99	0.9683	0.9679	0.00289
γ_e	N	0.9197	0.00164	0.01	0.99	0.9198	0.9197	0.00165
γ_m	N	0.9448	0.00103	0.01	0.99	0.9447	0.9446	0.00103
γ_x	N	0.8859	0.00225	0.01	0.99	0.8858	0.8856	0.00225
γ_{xd}	N	0.9751	0.00180	0.01	0.99	0.9753	0.9751	0.00181

$$\Sigma = \begin{pmatrix} 0.08642416 & -0.013786532 & -0.09287633 \\ -0.01378653 & 0.004669188 & -0.01586939 \\ -0.09287633 & -0.015869387 & 0.85507558 \end{pmatrix}$$



Estimation

Param.	Dist.	Prior				Posterior		
		Mean	Std	LB	UB	Mode	Mean	Std
ρ_q	U	1.5500	0.70083	0.10	3.00	0.9255	0.9231	0.00560
ρ_{qv}	U	1.5500	0.70083	0.10	3.00	0.8900	0.9023	0.06242
ε_q	B	0.5000	0.14142	0.00	1.00	0.3225	0.3216	0.02142
ε_w	B	0.5000	0.14142	0.00	1.00	0.4050	0.4136	0.07221
ε_m	B	0.5000	0.14142	0.00	1.00	0.1125	0.1156	0.02654
ρ_{c^*}	B	0.5000	0.14142	0.00	1.00	0.7100	0.6590	0.12524
$\rho_{\pi^{m^*}}$	B	0.5000	0.14142	0.00	1.00	0.8650	0.8501	0.04590
$\rho_{\pi^{rm^*}}$	B	0.5000	0.14142	0.00	1.00	0.5700	0.5536	0.07968
ρ_{zi^*}	B	0.5000	0.14142	0.00	1.00	0.3450	0.3463	0.09084
ρ_{tr^*}	B	0.5000	0.14142	0.00	1.00	0.8550	0.8508	0.04209
$\rho_{\pi^{c^*}}$	B	0.5000	0.14142	0.00	1.00	0.1650	0.1732	0.05215
$\rho_{z^{\pi q}}$	B	0.5000	0.14142	0.00	1.00	0.7900	0.6585	0.19998
$\rho_{z^{\pi w}}$	B	0.5000	0.14142	0.00	1.00	0.2300	0.2515	0.08583
$\rho_{\pi^{food}}$	B	0.5000	0.14142	0.00	1.00	0.2950	0.2916	0.08151
ρ_{z^x}	B	0.5000	0.14142	0.00	1.00	0.4050	0.4013	0.06727
ρ_{z^u}	B	0.5000	0.14142	0.00	1.00	0.8650	0.8371	0.07080
ρ_g	B	0.5000	0.14142	0.00	1.00	0.3250	0.3118	0.07713
ψ_x	B	0.5000	0.14142	0.00	1.00	0.4550	0.4483	0.06785

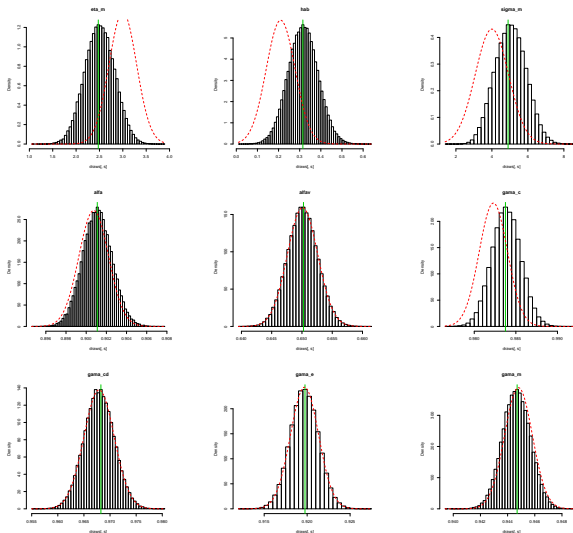


Estimation

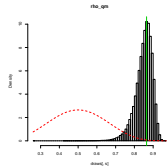
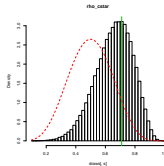
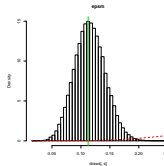
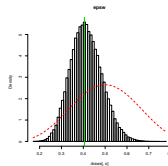
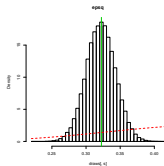
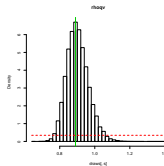
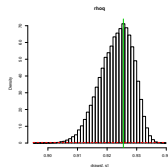
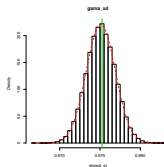
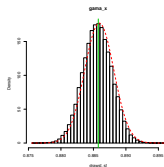
Param.	Dist.	Prior				Posterior		
		Mean	Std	LB	UB	Mode	Mean	Std
$\text{Var}(\pi_{food})$	IG	0.0005	100	1E-07	0.05	0.0004	0.0004	0.00008
$\text{Var}(c^*)$	IG	0.0005	100	1E-07	0.05	0.0000	0.0001	0.00001
$\text{Var}(g)$	IG	0.0005	100	1E-07	0.00	0.0000	0.0001	0.00001
$\text{Var}(\pi^*)$	IG	0.0005	100	1E-07	0.05	0.0001	0.0001	0.00003
$\text{Var}(q^m)$	IG	0.0005	100	1E-07	0.05	0.0005	0.0007	0.00016
$\text{Var}(q^{mr})$	IG	0.0005	100	1E-07	0.05	0.0040	0.0042	0.00097
$\text{Var}(tr)$	IG	0.0005	100	1E-07	0.05	0.0235	0.0252	0.00512
$\text{Var}(z^i)$	IG	0.0005	100	1E-07	0.05	0.0005	0.0005	0.00012
$\text{Var}(z^{ie})$	IG	0.0005	100	1E-07	0.05	0.0000	0.0000	0.00000
$\text{Var}(z^{\pi q})$	IG	0.0005	100	1E-07	0.05	0.0001	0.0001	0.00002
$\text{Var}(z^{\pi w})$	IG	0.0005	100	1E-07	0.05	0.0004	0.0007	0.00032
$\text{Var}(z^u)$	IG	0.0005	100	1E-07	0.05	0.0023	0.0045	0.00222
$\text{Var}(z^x)$	IG	0.0005	100	1E-07	0.05	0.0006	0.0006	0.00019



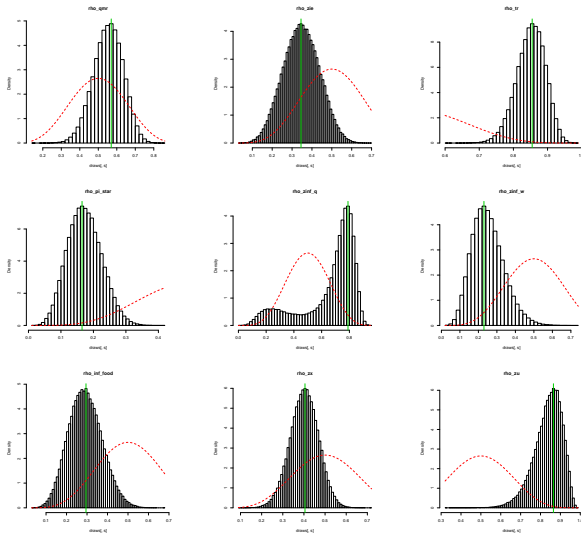
Priors - Posteriors



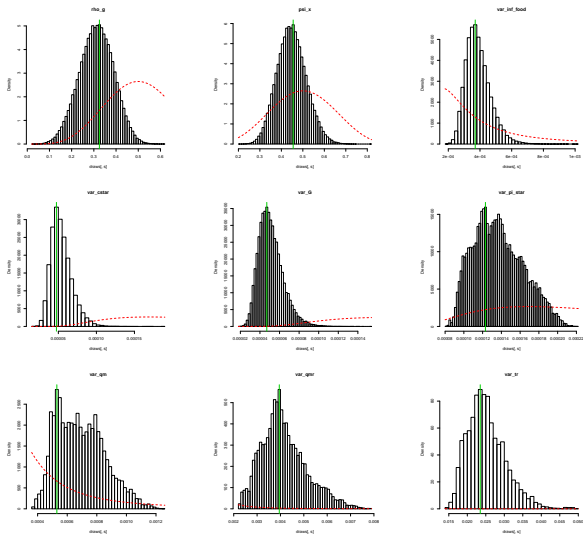
Priors - Posteriors



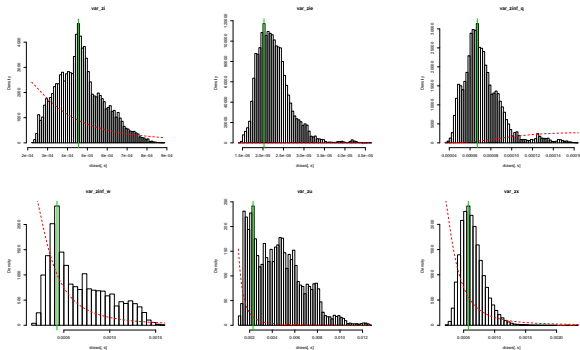
Priors - Posteriors



Priors - Posteriors



Priors - Posteriors



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Motivation

- Using a model as a main monetary policy forecasting tool is a very different exercise from simulating a model to answer particular questions.
- To forecast for policy, we need to match our model to as much of the useful information, even if that information comes in an awkward variety of shapes and forms.
- When forecasting with a DSGE model we have to take into account the following data problems:
 - ▶ Data uncertainty
 - ▶ Steady state uncertainty
 - ▶ Anticipated shocks that can be uncertain



Data Availability

Inflación							
Empleo							
	Salarios						
Tasa de interés mundial							
Producto mundial							
Remesas							

t

$t+2$



Pronósticos de otros modelos



Forecasting Method

- We use a state space representation of the model where the measurement equation relates the available data to the model variables and the transition equation contains the structure of the model.
 - ▶ The measurement equation allows us to include the uncertainty of the data through a time varying variance of the measurement error.
 - ▶ Also, it allows us to handle an unbalanced data set through the use of a selection matrix.
- The forecast of the model is a mix of the Kalman filter and the filter smoother.



State-space representation

The solution to the log-linearized first order conditions can be written as

$$\begin{aligned}c_t &= Gp_t & c_t &= GHp_{t-1} + G\epsilon_t \\ p_{t+1} &= Hp_t + \epsilon_{t+1} & p_t &= Hp_{t-1} + \epsilon_t.\end{aligned}$$

- Consequently, our transition equation is

$$x_t = \begin{pmatrix} c_t \\ p_t \end{pmatrix} = \begin{pmatrix} \mathbf{0} & GH \\ \mathbf{0} & H \end{pmatrix} \begin{pmatrix} c_{t-1} \\ p_{t-1} \end{pmatrix} + \begin{pmatrix} G \\ I \end{pmatrix} \epsilon_t$$

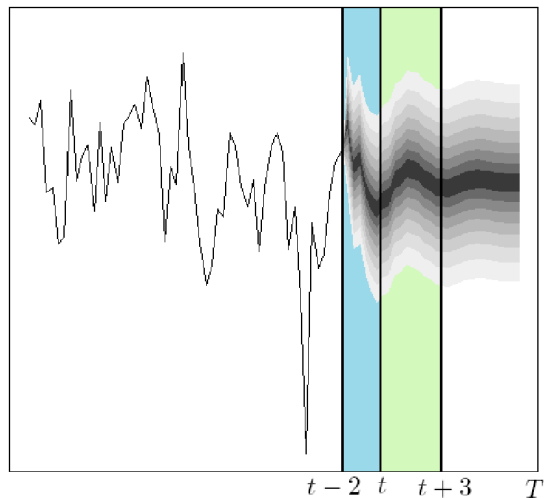
and our measurement equation is

$$W_t y_t = W_t I_S x_t + W_t \Gamma + W_t v_t$$

where W_t is a selection matrix and I_S is matrix where every row contains only one entry different from zero (=1) and every column has at most one entry different from zero. Γ is vector with the steady-state values.



Data uncertainty and off model information



- Observado con incertidumbre hasta t
- PCP de $t+1$ hasta $t+3$



Information about the future I

- Endogenous variables

This information is relevant for signal extraction. Which is the state of the economy?

Examples: surveys about expectations, forecasts from other models.

- 1 Include in the y_T vector the forecast and in x_T the expectations of the variable.

Modified measurement equation for time T (last period of the data set) that includes one period ahead information of the endogenous variable x_j

$$\begin{pmatrix} y_{1,T} \\ \vdots \\ y_{k,T} \\ f_t(y_{j,T+1}) \end{pmatrix} = \begin{pmatrix} x_{1,T} \\ \vdots \\ x_{k,T} \\ E_t(x_{j,T+1}) \end{pmatrix} + \begin{pmatrix} \zeta_{1,T} \\ \vdots \\ \zeta_{k,T} \\ \epsilon_f \end{pmatrix}$$

- 2 Calibrate the variance of ϵ_f to capture the uncertainty about the forecast.



Information about the future II

- Exogenous variables

This information is used to generate a conditional forecast that assumes a path for the exogenous variables.

Example: World GDP forecasts by the IMF

- In this case we include the value of the exogenous variable at time $T + h$ as an observable variable, consequently this information enters as a surprise in period $T + h$. At time T agents are not aware of the value of this variable at time $T + h$. The measurement equation for the “observable” exogenous variable x_l at time $T + h$ becomes

$$(y_{l,T+h}) = (x_{l,T+h}) + (v_{T+h})$$

where the variance of v determines the weight that $y_{l,T+h}$ has on the forecast.



Summary of I and II

Then we have a time varying measurement equation of the form:

$$\begin{aligned} \begin{pmatrix} y_{1,t} \\ \vdots \\ y_{k,t} \end{pmatrix} &= \begin{pmatrix} x_{1,T} \\ \vdots \\ x_{k,T} \end{pmatrix} + \begin{pmatrix} \zeta_{1,T} \\ \vdots \\ \zeta_{k,T} \end{pmatrix} && \text{for } t < T \\ \begin{pmatrix} y_{1,T} \\ \vdots \\ y_{k,T} \\ f_t(y_{j,T+1}) \end{pmatrix} &= \begin{pmatrix} x_{1,T} \\ \vdots \\ x_{k,T} \\ E_t(x_{j,T+1}) \end{pmatrix} + \begin{pmatrix} \zeta_{1,T} \\ \vdots \\ \zeta_{k,T} \\ \epsilon_f \end{pmatrix} && \text{for } t = T \\ (y_{l,t}) &= (x_{l,t}) + (v_t) && \text{for } T < t \leq T + H \end{aligned}$$



Forecasting method

Our forecasts for the variables contained in the y_t vector is

$$y_{t+h}^f = I_s x_{t+h}^f + \Gamma$$

with

$$x_{T+h}^f = \begin{cases} x_{T+h}^{T+H} & \text{if } t+h \leq T+H & \text{Smoother} \\ \Phi^{h-H} x_{T+H}^{T+H} & \text{if } t+h > T+H & \text{Standard Kalman filter forecast} \end{cases}$$

where $x_t^s = E[x_t | Y_s]$ and $Y_s = (y_1, \dots, y_t, \dots, y_s)$.

- $E[x_t | Y_s]$ is the Kalman smoother for $s > t$.
- $T + H$ is the last period for which there's at least data available for one variable in vector y_t .

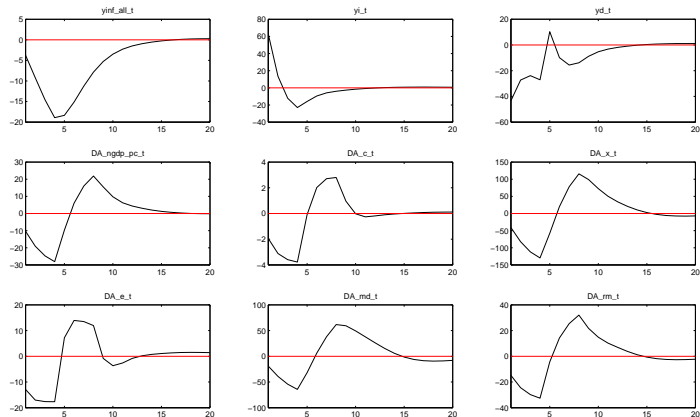


Outline

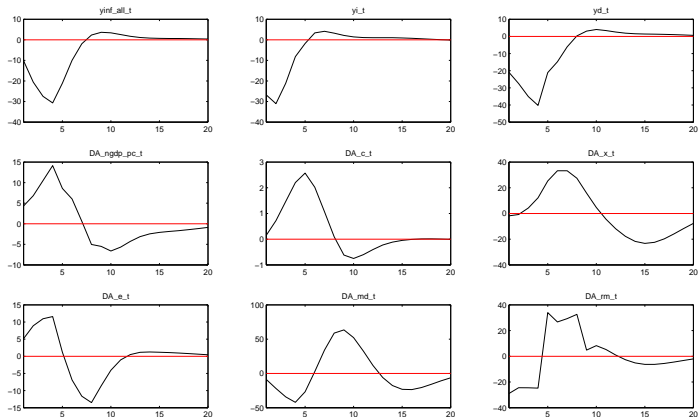
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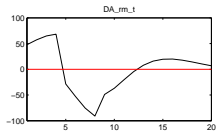
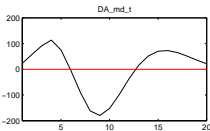
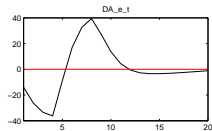
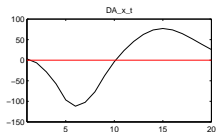
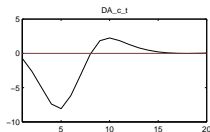
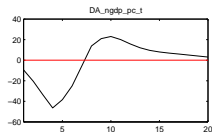
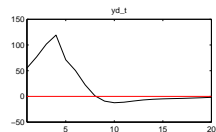
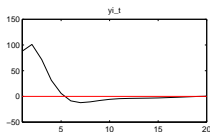
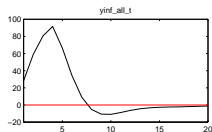
Monetary Shock



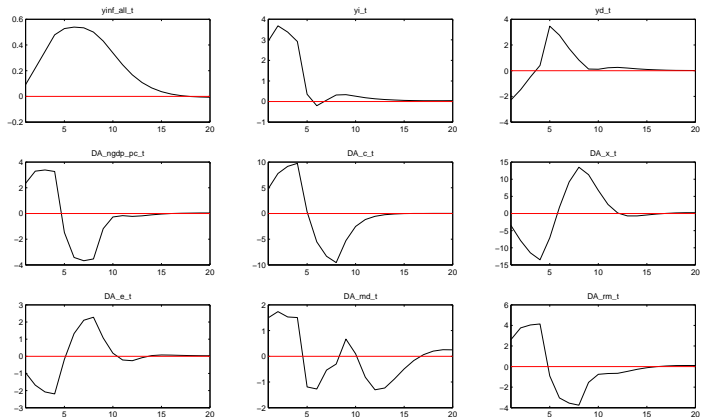
Productivity Shock



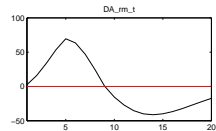
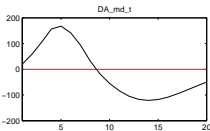
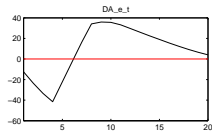
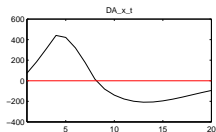
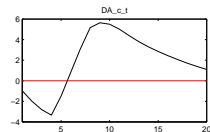
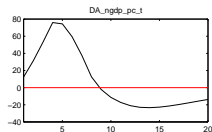
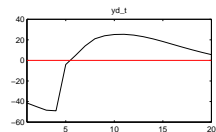
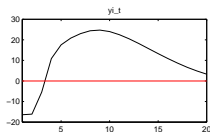
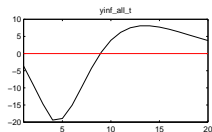
Cost Push Shock



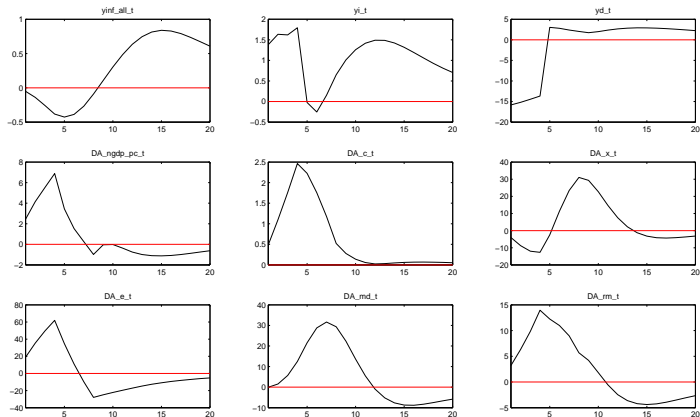
Preferences Shock



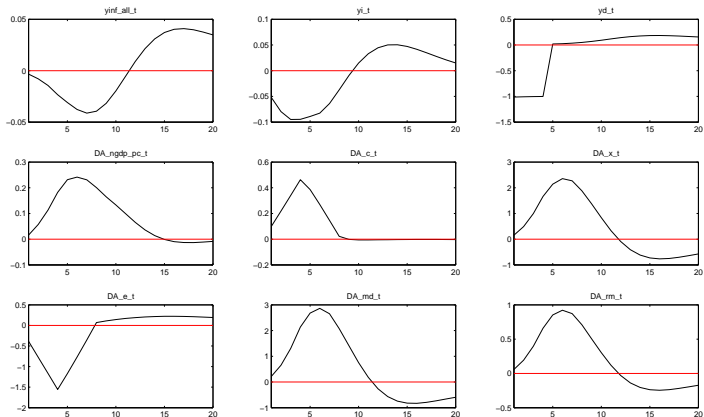
Investment Shock



External Demand Shock



Remittances Shock



Risk Premium Shock

